Breathing New Lifespan into HFC: Tools, Techniques, and Optimizations

Dr. Robert Howald
Fellow of the Technical Staff
ARRIS
June 11, 2013
Agenda

• Quantifying the “Why” of New Lifespan
  – Capacity Management Timeline
  – Spectrum Implications
• Capacity Optimization
  – QAM
  – FEC
  – OFDM
• Lifespan Implications
• Summary
Its All About Growth vs. Capacity

Traffic Growth vs Available HFC Capacity

- 50% CAGR
- 50% CAGR, Add IPV
- Max Consumption
- Max Consumption, N+0

Reference: Accounting For Techies: Taking it to the Ultra, Spring Technical Form Session 35

HFC Spectrum is Fully Utilized
The Growth of HD Programming and IP Data Continues
Capacity Management is Balancing Act of Services Evolution, Network Evolution, and Investment Timing
Keep an Eye on the Upstream

Return Path Lifespan vs CAGR

Assumes 80 Mbps Deployed (2013)
2x 64-QAM @ 6.4 MHz
1x 16-QAM @ 6.4 MHz

15 years
10 years
5 years

Recent CAGR Range

Growth Threat is Less, But Growth Mitigation Tools Limited
Critical to Act Ahead of Congestion Bottleneck
Peak Rate Considerations Play a Role
Spectrum Evolution and Expansion

Upstream
≥ 1 Gbps

Downstream
≥ 10 Gbps

1 GHz

New NG Forward

1.2 GHz

New NG Forward

Exact Crossover TBD

1.7 GHz

New NG Forward

More NG Forward
Power Load Burden of New D/S Spectrum

Example of Today’s 1 GHz Loading (with Analog)

~14 dB Tilt to 1 GHz

Expect to bridge RF power gap with technology (GaN)

Incur ~1 dB SNR Loss due to AM Optics Loading (flat)

Beyond a simple upgrade
- Architecture Evolution
- Power Consumption
- RF Payload Optimization
Physical Layer Evolution: Advanced M-QAM and New FEC
M-QAM Today & Tomorrow

- **64-QAM**
  - 28 dB SNR

- **256-QAM**
  - 34 dB SNR

- **1024-QAM**
  - 40 dB SNR

- **4096-QAM**
  - 46 dB SNR
  - +50% BW Eff

Shown at Equivalent BER of 1e-8 (No FEC)

Single-Carrier OR one Subchannel of OFDM

158 “channels”
(1 GHz of slots)
@4096-QAM
~ 10 Gbps Raw

“In-Betweens”:
512/2048-QAM

Bigger & Better?
8192/16384-QAM
How? The State of Modern HFC Delivery ....

Field Metrics Now Tell a Compelling Statistical Story About Bits Available to Mine

Architecture Analysis Projects the Range of SNR and Expected M-QAM Supported
How? The Power of Modern FEC

DVB-C2 ModCods vs SNR as simulated by ReDeSign

1024-QAM: 25 dB/27 dB/30 dB @ k/n = (75%, 83%, 90%)
4096-QAM: 32.5 dB/35 dB @ k/n = (83%, 90%)

Reference:
“Performance evaluation of advanced modulation and channel coding”
30 Nov 2009, ReDeSign – 217014

Legacy: Reed-Solomon
New: Low Density Parity Check (LDPC) Foundation

“Free” (wrt technical !) dB to the M-QAM Link Budget

Highest Code Rates

Now Snuggled Up
to Theoretical Capacity Bounds
How Hard is the FEC Working?

Delivered Error-Free!
Field Results + FEC Thresholds = Significant New Downstream Capacity

FEC Thresholds

Reference:

HFC Cascade CNR Threshold

~98% of CMs Measure > 32 dB
New Upstream Capacity vs. Band Split

Analog Return Performance Only
Digital Return – No BW Loading Loss (price of Gbps & ENOB)

1024-QAM Just Meets Threshold
2048-QAM Margin Insufficient

Analog or Digital Return (Similar typical performance)
Acceptable Robustness by Today’s Standards to 1024-QAM
2048-QAM Just Meets Threshold (Channel only, No HE Combining)

Note: Different LDPC-Based Code Family is Used Upstream
Orthogonal Frequency Division Multiplexing (OFDM)
The Role of OFDM

- **Capacity (High SNR Case)** \( \approx \frac{1}{3} \) [Bandwidth] \times [SNR(dB)]
  - More Bandwidth ⇒ **New Spectrum**
  - SNR Tool #1 ⇒ **Increase via Architecture:** FD, POE GW, Remote Phy
  - SNR Tool #2 ⇒ **Best Codes Get Most from SNR:** Update FEC

- **OFDM?**
  - Capacity Equation, **Long Form:** \( C \approx \frac{1}{3} \sum [\Delta f] [S(\Delta f) (\Delta BW) / N(\Delta f)]_{dB} \)
    - OFDM Subcarriers = \( \Delta BW \) (narrow subcarriers)
  - SNR not uniform over the entire Bandwidth
  - Minimize Capacity lost to variable channel conditions
    - Freq response, Micro-reflection, External interference

The Role of OFDM

Why OFDM?

- **Optimizes Channel Capacity**
  In particular with varying frequency response, multipath, and hostile interference channels

- **Granular spectrum allocation**
  Beneficial during band plan and service transitions

- **Multiple sources of supply** and cross-industry investment

- Consistency with **other standards** and cable network extensions (Home LAN, wireless, EPoC)

- **OFDM + LDPC** to Layer 1 as IP is to Layer 3 – likely final RF step (little more capacity worth exploiting)

Different Behaviors Against Common Impairments

**Impairment:** Narrowband Interference
CTB/CSO, Wireless Ingress, Shortwave & CB (Upstream)

**Mitigation:** Bit Loading, Avoidance/Muting (if necessary), Frequency Domain Interleaving of Subcarriers

**Impairment:** Burst and Impulse Noise

**Mitigation:** OFDM Parameters, FEC Strength, Time-Domain Interleaving, Bit Loading (freq dependent noise)

**Impairment:** Carrier Frequency and Phase Error

**Mitigation:** Dedicated OFDM Subcarriers as Pilots for Acquisition, Phase Estimation, and Tracking
Net Capacity Benefits and HFC Lifespan Accounting
Its All About Growth vs. Capacity

Bandwidth Efficiency and Spectrum Availability Together Pay Big Dividends
“Spectrum” = Altogether New and Reclaimed from Inefficient Legacy
With CAGR Settling, Could be the Difference in Long-Term Sustainability
Without Settling, CAGR Always Wins – We Can Only Impact When
Summary

• Continued Pressure to Manage the Aggressive IP Traffic Growth and Increased HD Programming
• More Capacity is Needed to Manage the Growth and Extend the Lifespan of HFC
• HFC Channels Deliver Relatively High SNR and Operate Well Below Their Capacity Potential—Downstream and Upstream
  – Tools Exist to Extract this Capacity
  – More Spectrum is Available on the Coaxial Medium than is Used Today
  – Architecture Evolution contributes to Better Capacity Exploitation
• Monitoring CAGR Trends is Key to Evolution Timing and Investment
  – A Reasonable Window of Time Exists to Assess Trends
• All is Quantifiable: New Capacity and its Implications to Lifespan
  – Significant New Capacity and the Added Lifespan Shown
  – Can Project the Sustainable Scenarios for HFC
Thank You!

rob.howald@arrisi.com
HEVC in a Changing World—What Can MSOs Expect?

Mukta Kar, CableLabs
Yasser Syed, Comcast Labs
Munsi Haque, Consultant

June 3, 2013
Outlines

• Introduction – What is HEVC?
• What is different from AVC?
• Why do we need another non-backward compatible video codec?
• Performance of HEVC
• Changes the perspective of competition
  – What does it mean to cable?
• What can MSOs take advantage of?
• Conclusions
What is HEVC?

• A 3rd generation video compression standard.
• HEVC = 2:1 compression gain over AVC or = 4:1 compression gain over MPEG-2
• Added more complexity; Moore’s law enables HEVC
• HEVC will enable delivery of 4kTV and beyond with higher quality level
• Cable can deliver, Others? Cable = Best network + Most Bandwidth
• Huge benefits to consumers
  – A lot of high quality content to watch, at home and on the go
Another non-backward compatible video codec?

• Internet video uses over 50% of the BW and is growing. The new services Cloud DVR, TV everywhere, 4kTV will need even more BW
• Growing demand for video services on subscribers’ other devices
• Cable is moving to IP delivery to replicate broadcast functionalities.
• We are in a bandwidth crunch
• Old codecs prevents cable from cashing new services
What is different from AVC?

- HEVC is an enhanced “AVC” + a few new tools
- Added parallelization tools:
  - Enables parallel encoding and decoding in a multiprocessor GPU; will help prolong battery life
  - Helps in encoding and decoding of large resolution pictures such as 4kx2k and 8kx4k
HEVC Block Diagram

Input Video Signal

Split into CTUs

General Coder Control

Transform, Scaling & Quantization

Scaling & Inverse Transform

Quantized Transform Coefficients

Intra Prediction Data

Filter Control

Analysis

Coded Bitstream

Header Formatting & CABAC

Decoder Output Video Signal

Decoded Picture Buffer

Motion Data

Motion Estimation

Motion Compensation

Intra/Inter Prediction

Intra-Picture Estimation

Intra-Picture Prediction

Filter Control Analysis

Deblocking & SAO Filters

General Control Data

Source: G. Sullivan, Microsoft
## Major Differences Between MPEG-2, AVC, and HEVC

<table>
<thead>
<tr>
<th>Coding Tools</th>
<th>MPEG-2</th>
<th>AVC</th>
<th>HEVC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intra-prediction</strong></td>
<td>None</td>
<td>Yes (9 modes)</td>
<td>Yes (35 predictions)</td>
</tr>
<tr>
<td><strong>Inter-prediction</strong></td>
<td>Yes (No B-picture as reference)</td>
<td>Yes (allows b-picture as reference)</td>
<td>Same as AVC</td>
</tr>
<tr>
<td><strong>CU size</strong></td>
<td>16x16 (fixed) Macroblock</td>
<td>16x16 (same as MPEG-2 video)</td>
<td>Variable, 64x64, 32x32, 16x16, and 8x8</td>
</tr>
<tr>
<td><strong>PU size</strong></td>
<td>16x16</td>
<td>16x16, 16x8, 8x16</td>
<td>32x32, 16x16, 16x8, 8x16 8x8, 8x4, 4x8, 4x4</td>
</tr>
<tr>
<td><strong>TU size</strong></td>
<td>8x8 (DCT floating point)</td>
<td>8x8 and 4x4 (DCT integer)</td>
<td>32x32, 16x16, 8x8, 4x4 (DCT integer and 4x4 DST integer)</td>
</tr>
<tr>
<td><strong>In-loop filter</strong></td>
<td>None</td>
<td>One Deblocking filter</td>
<td>Two in-loop filters (deblocking and SAO)</td>
</tr>
<tr>
<td><strong>Entropy</strong></td>
<td>VLC</td>
<td>CAVLC and CABAC</td>
<td>CABAC only</td>
</tr>
<tr>
<td><strong>Parallel Processing</strong></td>
<td>None</td>
<td>None</td>
<td>Tile and Wavefront</td>
</tr>
</tbody>
</table>
# Performance and Complexity

<table>
<thead>
<tr>
<th>Resolution/FPS</th>
<th>HEVC</th>
<th>AVC</th>
<th>MPEG-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>UHDTV- 4K/60</td>
<td>8-15 Mbps</td>
<td>18-22 Mbps</td>
<td>High</td>
</tr>
<tr>
<td>HD- 1080/720P60</td>
<td>1.5-3.5 Mbps</td>
<td>5-9 Mbps</td>
<td>9-15 Mbps</td>
</tr>
<tr>
<td>HD- 720p30</td>
<td>0.8-2.0 Mbps</td>
<td>1.5-4 Mbps</td>
<td>3-5 Mbps</td>
</tr>
<tr>
<td>SD</td>
<td>0.4-0.7 Mbps</td>
<td>0.7-1.5 Mbps</td>
<td>2-3 Mbps</td>
</tr>
</tbody>
</table>

## Performance Stats

<table>
<thead>
<tr>
<th>Performance Stats</th>
<th>HEVC over AVC [High Profile]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encoder Complexity</td>
<td>~10x (or less)/Early 4x</td>
</tr>
<tr>
<td>Decoder Complexity</td>
<td>~1.4x</td>
</tr>
<tr>
<td>Memory</td>
<td>~1.25x</td>
</tr>
<tr>
<td>Memory Bandwidth</td>
<td>~1.25x</td>
</tr>
</tbody>
</table>

Source: A. Rodriguez, Cisco
HEVC HM 8 PSNR Performance

Average over entire test set and all bit rates:

<table>
<thead>
<tr>
<th>Encoding</th>
<th>H.264 / MPEG-4 AVC HP</th>
<th>H.263 CHC</th>
<th>MPEG-4 ASP</th>
<th>H.262 / MPEG-2 MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEVC MP</td>
<td>40.3 %</td>
<td>67.9 %</td>
<td>72.3 %</td>
<td>80.1 %</td>
</tr>
<tr>
<td>H.264/MPEG-4 SVC HP</td>
<td>–</td>
<td>46.8 %</td>
<td>54.1 %</td>
<td>67.0 %</td>
</tr>
<tr>
<td>H.263 CHC</td>
<td>–</td>
<td>–</td>
<td>13.2 %</td>
<td>37.4 %</td>
</tr>
<tr>
<td>MPEG-4 ASP</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>27.8 %</td>
</tr>
</tbody>
</table>

Source: G. Sullivan, Microsoft
HEVC can reduce the complexity and costs of handling multiple streams over AVC; as such benefits
– OTT services
– Enables mobile/wireless delivery
– Backhaul can save bandwidth, enabling IP fiber delivery
– Saves Storage
– Single stream can support dynamic trickplay
– Decreases service bandwidth for each device
– Indirectly can assist in migration to IP until multicast becomes practically deployable
Should MSOs adopt

• HEVC adoption enables MSOs to launch new services
• HEVC will enable premium quality high resolution video delivery (4kTV)
• MSOs are in the process of migrating to IP
  – This will enable easier HEVC adoption
• HEVC defined one Profile, results in device convergence
• One profile = significant reduction in number of streams
• Consumers, service providers and studios will benefit greatly from this convergence
Conclusions

- HEVC is a new compression standard with 2:1 compression gain over AVC, example
  - 100 MPEG-2 video channels = 200 AVC ch = 400 of HEVC ch
- It will enable delivery 4kTV & 8kTV; a plus for cable
- Moore’s law makes HEVC complexity a non-issue
- Migration to IP will minimize the issue of its non-backward compatibility
- One profile translates into single platform convergence, benefits all stakeholders in the value chain, especially the consumers.
- HEVC brings a huge opportunity to deliver more and better video services to consumers
A Simple Approach for Deriving the Symbol Error Rate of Non-Rectangular $2^{2k+1}$ M-ary AMPM Modulation

Session 28:
Modulation, Multiplexing and Moore: Approaches for Cable’s Brave New Network World

Tuesday, June 11, 2013
2:00 PM – Room 207A

Patricio Latini, Vice President of Technology CALA – ARRIS
Coauthor: Ayham Al-Banna, CTO Office - ARRIS
Overview of the Paper

• Odd-bit QAM constellations are currently used in many applications like DOCSIS, MPEG Video and future DOCSIS.

• Some of these applications use odd-bit QAM constellations that are not arranged in rectangular fashion.

• Rectangular even-bit QAM constellations have been widely analyzed.

• A expression for the exact probability of symbol error in the presence of AWGN has already been analyzed and established for even-bit constellations.
Even and odd-bit Constellations

- **64-QAM**
- **8 QAM**
Symbol Error Rate Waterfall Plots

Symbol Error Probability

Eb/No (dB)
Overview

- Even Constellations well-known exact SER expression

\[ P_{SE} = 4 \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3\varepsilon_{av}}{(M - 1)N_0}} \right) - \left( 2 \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3\varepsilon_{av}}{(M - 1)N_0}} \right) \right)^2 \]

- Odd Constellations only had an upper bounded expression

\[ P_e < 4 \left( 1 - \frac{3}{2M} \right) Q \left( \sqrt{\frac{6 \log_2(M) \varepsilon_b}{2M - 1 \frac{N_0}{N_0}}} \right) \]
Derivation of the Exact Expression

- 4 Types of Symbols
- Calculated the individual Pe contribution of each type.
- Accumulated all the contributions and created general expression
SER Approximation Error

\[ P_e = \left( 4 - 8 \sqrt{\frac{1}{2M} + \frac{2}{M}} \right) Q \left( \sqrt{\frac{6 \log_2(M) \varepsilon_b}{(2M-1) N_0}} \right) + \left( 8 \sqrt{\frac{1}{2M} - 4} \right) Q^2 \left( \sqrt{\frac{6 \log_2(M) \varepsilon_b}{(2M-1) N_0}} \right) + \left( 4 \sqrt{\frac{1}{2M} - 4} \right) Q \left( \sqrt{\frac{12 \log_2(M) \varepsilon_b}{(2M-1) N_0}} \right) \]
SER Approximation Error

- The derived expression is below the upper bounded.
- Verified that according to the theory the upper bounded expression error growths in low SNRs.
SER MATLAB Simulation

- Verified with MATLAB simulation that the derived expression was in complete agreement with a system simulation.
Conclusions

• The usage of an exact expression for odd-QAM modulations is very beneficial in order to derive exact SNRs from field obtained SER.

• The obtained expression gives better results than the approximate upper bounded in low SNRs.

• The exact expression has been verified with system simulations.
Thank You

Patricio Latini
Vice President of Technology CALA
ARRIS
Smart ABR (SABR):
The Future of Managed IP Video Services

John Ulm
Fellow of the Technical Staff
ARRIS - CTO Architecture Team
john.ulm@arrisi.com

Co-authors:
Ajay Luthra, Praveen Moorthy,
Mark Schmidt, Haifeng Xu
Smart ABR (SABR)
The Future of Managed IP Video Services

• ABR Introduction
  – Potential Issues
  – Potential Solutions

• Lab Results
  – Unmanaged ABR
  – DOCSIS QoS
  – SABR

• Conclusions
ABR Introduction

At startup and when changing bit rates, ABR enters Buffering Stage

Chunk Bit Rates change based on client’s perceived bandwidth availability
Potential Issues with ABR Delivery

- Instability
  - QoE Impacts

- Unfairness
  - Multiple types

- Inefficiencies

- No Admission Control

Challenge – How do we offer a Managed Video Service to ABR Devices with required QoE?
Potential Solutions

• DOCSIS QoS
  – Configure Service Flow for every ABR stream

• Smart ABR (SABR) – Cloud Based Server solution
  – Centralize Chunk Bit Rate selections in the cloud
    • Take control away from client
    • Clients remain unchanged, simple
  – Intelligent Bit Rate selection
    • Provide appropriate Bit Rate based on video stream needs
      – Screen size, Subscriber tiers, premium channels, video complexity
SABR Lab Test Setup

Unmanaged Adaptive Bit Rate Streaming

**Test Configuration 1:**
- DS BE MSR: 12 Mbps

**Test Configuration 2a:**
- DS 480p30 SF MSR: 2.5 Mbps
- DS 720p30 SF MSR: 4.5 Mbps

**Test Configuration 2b:**
- DS 480p30 SF MSR: 5.0 Mbps
- DS 720p30 SF MSR: 9.0 Mbps

MSR: Maximum Sustained Traffic Rate
DS: Downstream
US: Upstream
BE: Best Effort
SF: Service Flow
Unmanaged ABR Results

~77% Channel Utilization
Unmanaged ABR Results

Significant Instability and Unfairness
ABR Results with DOCSIS QoS

Almost 80% Channel Utilization
ABR Results with DOCSIS QoS

Stability improvements but Unfairness Continues
SABR Results

Aggregate Bit Rate - Requested & Used

84%+ Channel Utilization
SABR Results

Bit Rate Provided

Estimated Buffer Depth

Solid stability but is it Fair?

Bit Rate changes intentional to improve QoE
QoE Results – PSNR CDF

Unmanaged ABR

ABR + DOCSIS QoS

PSNR – Peak Signal to Noise Ratio
QoE Results – PSNR CDF

Unmanaged ABR

SABR with 15Mbps Cap

PSNR – Peak Signal to Noise Ratio
Unmanaged ABR, DOCSIS QoS Conclusions

• Managed IP Video Service must address ABR QoE
  – Instability, Unfairness, Inefficiencies, Congestion control

• DOCSIS QoS
  – Requires a Service Rate that is twice max Chunk Bit Rate
  – Provides some benefits but does not fix problems
    • Three levels of congestion control: DOCSIS, TCP, ABR
  – Potential scaling issues using PCMM
  – Not applicable to other network types: wireless, FTTP
Smart ABR (SABR) Conclusions

• SABR centralizes Chunk Bit Rate selections in the cloud
  – Provides Stability, Fairness, Improved Channel Utilization
  – Graceful degradation during congestion

• Intelligent Bit Rate selection
  – Improved QoE
  – Potentially 30-50% Stat Mux gains
    • Tests show gain from 23.6Mbps to 13.5Mbps

SABR – provides operators with increased video capacity while maintaining consistent QoE across all clients
4K-QAM, LDPC, OFDM for Gbps Data Rates

Dave Urban

June 11, 2013
<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guard Interval (Cyclic Prefix)</td>
<td>$T_g$</td>
<td>2.5</td>
<td>μs</td>
</tr>
<tr>
<td>Useful Symbol Time</td>
<td>$T_u$</td>
<td>40</td>
<td>μs</td>
</tr>
<tr>
<td>subcarrier frequency spacing</td>
<td>$\delta f$</td>
<td>25</td>
<td>kHz</td>
</tr>
<tr>
<td>FFT sampling frequency</td>
<td>$R_{\text{sampling}}$</td>
<td>204.8</td>
<td>MHz</td>
</tr>
<tr>
<td>FFT size</td>
<td>$N_{\text{FFT}}$</td>
<td>8,192</td>
<td>subcarriers</td>
</tr>
<tr>
<td>channel width</td>
<td>CW</td>
<td>190</td>
<td>MHz</td>
</tr>
<tr>
<td>available subcarriers</td>
<td>$N_A$</td>
<td>7,600</td>
<td>subcarriers</td>
</tr>
<tr>
<td>pilot spacing</td>
<td>$\delta_{\text{pilots}}$</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>number of pilots</td>
<td>pilots</td>
<td>60</td>
<td>subcarriers</td>
</tr>
<tr>
<td>data subcarriers</td>
<td>$N_{\text{data}}$</td>
<td>7,540</td>
<td>subcarriers</td>
</tr>
<tr>
<td>constellation points</td>
<td>$M$</td>
<td>4096</td>
<td>points</td>
</tr>
<tr>
<td>data subcarrier bit loading</td>
<td>$b$</td>
<td>12</td>
<td>bits</td>
</tr>
<tr>
<td>FEC information bits</td>
<td>$K_{\text{BCH}}$</td>
<td>14,232</td>
<td>bits</td>
</tr>
<tr>
<td>FEC codeword size</td>
<td>$N_{\text{LDPC}}$</td>
<td>16,200</td>
<td>bits</td>
</tr>
<tr>
<td>data rate</td>
<td>$R_{\text{data}}$</td>
<td>1,870.3</td>
<td>Mbps</td>
</tr>
<tr>
<td>signal to noise ratio threshold</td>
<td>SNR</td>
<td>35.23</td>
<td>dB</td>
</tr>
</tbody>
</table>
4.5 µs 1925 ft P3 875 Attenuation

Frequency in MHz

Attenuation in dB

-40
-30
-20
-10
0

0 100 500 1000 1500 2000

frequency in MHz

a = 0.0389899
f = 0.0001888f
Parity Check Matrix
1120 variable nodes
280 check nodes

\[ H \cdot c^T = 0 \]
\[ r_{ji}^l(0) = \frac{1}{2} + \frac{1}{2} \cdot \prod_{i' \in V \setminus j} \left( 1 - 2 \cdot q_{i'j}^{l-1}(1) \right) \]
\[ q_{ij}^l = l_i + \sum_{j' \in C_{j|i}} r_{j'i}^{l-1} \]
COMCAST